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# THE WORST-CASE MATHEMATICAL THEORY OF SAFE-ARMING

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Denis A. Silvia

May 1984

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT CENTER
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The safety and reliability of in-line safe/arming devices is analyzed quantitatively by a three step process: a generalized definition of the safe/arming function is developed, an analytical methodology to implement the safe/arm definition is constructed, and some proposed in-line devices are evaluated using the quantitative methodology.

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# I. INTRODUCTION

A conventional explosive train consists of a fuze, detonator, safe/arming* (s/a) mechanism, booster, and warhead. The s/a mechanism is interposed between the detonator and booster to protect the main explosive charge from accidental detonation of the sensitive primary explosive in the detonator (see Figure 1). Historically, the approach used in s/a devices has been the "out-of-line" method. The detonator is separated from the booster by one or more physical barriers. Accidental detonation of the detonator cannot penetrate the barrier(s) and so will not cause detonation of the warhead.

Although this method is simple and direct, some deficiencies have long been noted, including quality assurance problems, insufficient reliability, and vulnerability to environmental degradation. To combat these deficiencies, alternative safing methods, so-called "in-line" devices, have been proposed to eliminate the out-of-line mechanisms in conventional weapons. Proposed devices can be partitioned into two categories: high-power devices and low-power devices.

In the high-power method the primary explosive detonator is replaced with one that has no primary explosive at all. Instead, the system uses a very high-power electrical supply capable of detonating booster explosive directly. The detonator can thus be placed in direct line with the main charge. The safe/arm function is not eliminated, since the high-power electrical supply must be isolated from the warhead by an electrical s/a device (see Figure 2). The high-power approach requires cost and volume allocations which may not be available.

The low-power method uses a set of detonators which contain primary explosive. When the fuze makes the decision to detonate the warhead, it generates an electrical code. The code is sent to the set of detonators which converts the electrical code into a set of (perhaps sequenced) detonations. The s/a device then examines the coded set of detonations and determines if the code is valid. If so, the warhead is detonated. If not, the system duds (see Figure 3).

Although it is currently in a more advanced stage of development, the high-power method follows the general approach used in nuclear devices where safety and reliability are important, but cost and volume limitations are not as severe as they are in conventional weapons. This report will discuss only the safety/reliability requirements of low power s/a devices, but some of the discussion is relevant to analysis of s/a mechanisms in general.

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*Safe/arming, safe/arm, and s/a are used interchangeably in this report.

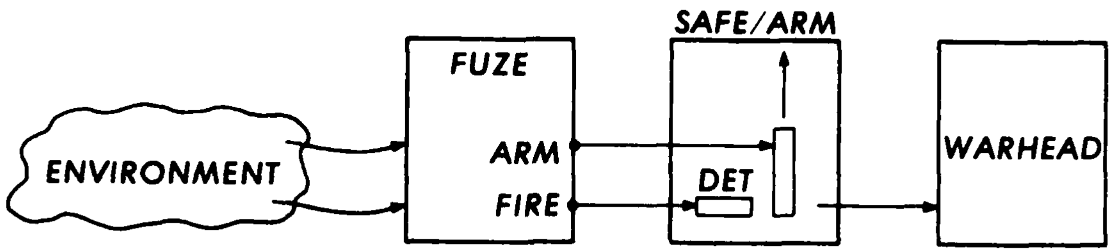


Figure 1. A Conventional Explosive Train

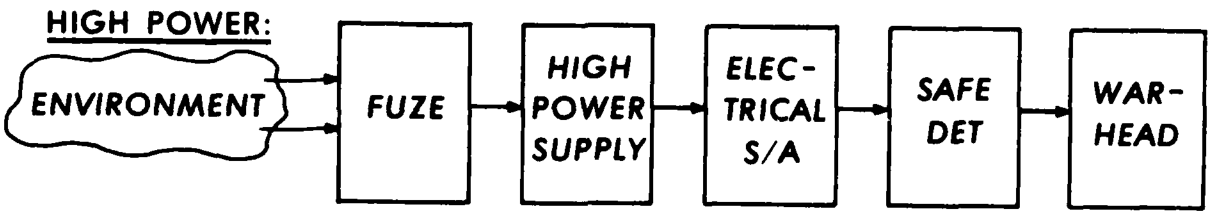


Figure 2. The High Power Method of In-Line Safe/Arming Devices

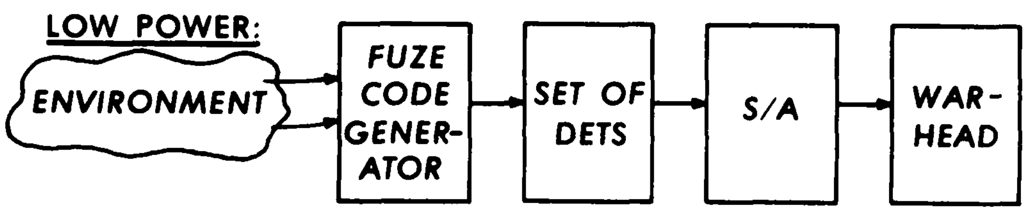


Figure 3. The Low Power Method of In-Line Safe/Arming Devices

## II. OBJECTIVES

The work reported here was undertaken in support of the Explosive Logic Technology project at the Ballistic Research Laboratory (BRL). Its purpose was to quantitatively define the requirements, limitations, and utility of applying explosive logic to the design of in-line safe/arming systems. Of particular interest was the analysis of those in-line safe/arm devices currently being investigated at BRL and other laboratories.

## III. GENERALIZING THE SAFE/ARMING PROCESS

When we examine Figures 1-3, it becomes clear that the traditional safe/arm mechanism - a mechanical barrier between detonator and warhead - must be generalized to include the new kinds of systems. The environment acts as a source of information. The fuze interprets this information. When the fuze determines that the information it has received warrants action, it sends commands to the s/a device. The safe/arm device must examine these commands to make sure that they come from a legitimate fuze decision rather than from some stray environmental signal. From this viewpoint, we can state a general definition of the safe/arming process.

THE SAFE/ARMING PROCESS IS ONE OF VALIDATING
A FUZE ORDER TO DETONATE THE WARHEAD.

The power of this simple generalization becomes apparent when we examine just how the s/a device goes about conducting its validation. The s/a device decision is a conceptually simple one - either the fuze ordered an action or it did not. However, when we impose stringent reliability and safety requirements on the results of the s/a device decision, then the validation process becomes difficult. The complexities added by safety and reliability criteria are illustrated in Figure 4, which shows the interpretation of the s/a device decision as a statistical process: If the s/a device accepts the $H_o$ hypothesis (fuze order is correct), then it risks an error of the second kind - that the fuze order was not actually correct. Since the whole purpose of the s/a device is to avoid this kind of mistake, the standards for safety (error of the second kind) are high:

THERE SHOULD BE NO MORE THAN ONE
CHANCE IN ONE MILLION THAT THE
S/A METHOD WILL DETONATE THE WARHEAD ON A FALSE SIGNAL.

If the s/a device tries to meet its safety requirements by rejecting $H_o$, then it is accepting the alternate hypothesis (that the code is invalid), $H_1$. Accepting $H_1$ when, in fact, the code WAS valid is an error of the first kind - the reliability problem. Reliability standards are less well defined

than safety standards for s/a devices, but we can develop one of our own by noting that users are unlikely to want a complicated s/a method that is less reliable than a simple one. Although compromises with this rule on reliability are often forced upon designers, it is a useful guide. To state this formally:

THE DETONATOR RELIABILITY OF ANY S/A SYSTEM

SHOULD BE AS HIGH AS THE DETONATOR

RELIABILITY OF A SINGLE DETONATOR SYSTEM.

Using the generalized concept that the s/a process is a validation step between fuze and warhead, the systems of Figures 1-3 can be combined into the conceptual organization shown in Figure 5. Here the explosive train is viewed as a mathematical system. The fuze, whose function is to gather and interpret environmental information and make the detonation decision, represents the input code (or object language) which is validated by the s/a device.

The input code is transformed into a set of binary variables at the beginning of the s/a step - whether this step is called part of the fuze or part of the s/a process is arbitrary. Even if the input quantities are measurable variables, such as set back force or spin rate, thresholds are introduced to make the quantities binary. This is necessary in any type of s/a process because of the yes/no decision that the s/a device has to make.

The safe/arming step (syntax language) consists of manipulating the input code in order to determine if the object language constitutes a valid fuze order. In effect, the s/a process proves a "theorem" by detonating the warhead or proves a "contradiction" by going dud. In order for any safe/arm device to perform its mission to the levels of safety and reliability required, it is clear that the input code must contain enough information for the decision to be made. We can state this as a formal conclusion:

EVEN IF THE S/A "HARDWARE" WORKS PERFECTLY,

IT CANNOT EXCEED THE LIMITATIONS OF THE OBJECT LANGUAGE.

THE INPUT CODE MUST

CONTAIN SUFFICIENT INFORMATION TO MEET

BOTH SAFETY AND RELIABILITY REQUIREMENTS.

## IV. A MATHEMATICAL APPROACH TO SAFE/ARMING ANALYSIS

### A. S/A Reliability

As we saw in the previous section, analysis of the s/a process has to begin with the analysis of a code of sequenced binary pulses. If the binary code sources are detonators, then the reliability criterion requires that the system detonator reliability must equal or exceed the reliability of a single detonator. This immediately excludes any system of N detonators in series ($N > 1$) where all N must function, since the reliability of N detonators is less than a single one. We can use the same criterion to examine the reliability of other systems, such as [(N-1)/N] where all but one must function, or [(N-2)/N] where all but two must function, etc. The easiest way to do this is to use the binomial expression

$$R[(N-k)/N] = \sum_{s=0}^{k} \binom{N}{s} r^{N-s}(1-r)^s \geq r \tag{1}$$

where R[(N-k)/N] is the reliability of an [(N-k)/N] system, and r is the reliability of a single detonator. If the first few terms of Equation (1) are written explicitly, we obtain

$$R[(N-k)/N] = r^N + Nr^{N-1}(1-r) + \frac{N(N-1)}{2} r^{N-2}(1-r)^2 + \ldots \tag{2}$$

Each type of s/a process, which we will call a <u>class</u>, requires a different number of terms of the binomial expansion shown in Equation (2). Thus the

[N/N] class requires only the first term
[(N-1)/N] class requires the first two terms
[(N-2)/N] class requires the first three terms, etc.

Each class has its own reliability equation of the form

$$f(r) \geq r\,. \tag{3}$$

At the point where f(r) just equals r we can write

$$f(r) - r = 0\,. \tag{4}$$

The reliability equations for the first three classes can then be written

$$\text{N/N:} \qquad r^N - r = 0 \tag{5a}$$

$$\text{(N-1)/N:} \qquad r^N + Nr^{N-1}(1-r) - r = 0 \tag{5b}$$

$$\text{(N-2)/N:}\ r^N + Nr^{N-1}(1-r) + \frac{N(N-1)}{2}r^{N-2}(1-r)^2 - r = 0\ . \tag{5c}$$

Figure 6 shows the behavior of these polynomial functions for different values of r.

Although detonators with reliability of .9999 have been built, reliability is a function of cost. If a practical limit for detonator electrical reliability is .99, then the limiting number of detonators in a [(N-1)/N] class system is 15. In other words, if we can't expect the detonator to be more than 99% reliable, then any [(N-1)/N] system with more than 15 detonators will not meet the reliability standard. The upper limit for a [(N-2)/N] system is 44. These values were calculated directly from Equations (5b) and (5c). The task is to try to meet the safety requirement within the limits set by reliability needs.

## B. S/A Safety

System safety is more complicated to analyze. Since the purpose of a s/a device is to protect against an accidental explosive event, it is not sufficient to assume a random (unbiased) environment. For any s/a system we can write

$$P\left\{\text{System Event}\right\} = \sum_{\substack{\text{ALL}\\ \text{STRESS}}} P\left\{\begin{matrix}\text{Fail}\\ \text{Stress}\end{matrix}\,\middle|\,\begin{matrix}\text{See}\\ \text{Stress}\end{matrix}\right\} \times P\left\{\begin{matrix}\text{See}\\ \text{Stress}\end{matrix}\right\}, \tag{6}$$

where P (See Stress) is the probability that the system is subjected to such stress and P (Fail Stress) is the probability that the system fails under such a stress. A system event occurs when the safe/arm device directs the warhead to explode. This equation defines the necessary and sufficient conditions for s/a safety. Unfortunately, it is rarely possible to know any of the terms in Equation (6). In order to simplify the equation, suppose we assume that the system will always see the worst possible stress. Then we can write

$$P\left\{\begin{matrix}\text{System}\\ \text{Event}\end{matrix}\right\} \leq P\left\{\begin{matrix}\text{System Fails}\\ \text{Worst Stress}\end{matrix}\right\} \times 1\ . \tag{7}$$

This is much stronger than the necessary and sufficient condition, but it does achieve simplification of the unknown terms. For a specified system we can try to determine how the application of the worst possible conditions will affect the performance of the s/a device. Since the stress will be applied through the individual detonators, each of which must perform correctly, we can write

$$P\left\{\text{System Event}\right\} \leqslant S\left[P\left\{\begin{matrix}\text{Individual Det. Fails}\\ \text{Under Worst Stress}\end{matrix}\right\}\right] < 10^{-6}, \tag{8}$$

where S is a strategy function determined by the structure of the system. This is clearly a worst-case assumption, and one might legitimately argue that it is too severe. The issue ultimately devolves into an explicit question: Do we design for the worst possible situation or for something less? The answer to the question is beyond the scope of this report.

If the value of S can be held below the safety criterion (one failure/million trials), then we are assured that the s/a system will meet the safety criterion AS FAR AS DETONATOR SAFETY IS CONCERNED. The capitalization is used to emphasize that design weaknesses that permit environmental stress to "sneak around" the s/a device logic and detonate the warhead are a separate problem. This analysis only examines detonator strategies.

## V. ENVIRONMENTAL STRESS

In the preceding section, we developed an approach to s/a device analysis which assumed that the worst possible environmental conditions will be experienced by a s/a device. The approach requires stipulation of two parameters: the stress imposed by the environment and the system response to that stress.

### A. Partitioning Environments by Range of Application

Environmental stresses can be characterized by their range of application. If a stress can "reach in" and exercise an individual detonator, or proper subset of detonators, then we shall call it "Local." If a stress must be applied more-or-less simultaneously and equally to all the detonators (system as a whole), then we will call it "Global."

### B. Partitioning Environments by Intelligence

Environmental stresses can also be characterized by whether or not they are "planned." If a stress is deliberately applied in intensity and timing, then we will call it "Intelligent." If its intensity and timing are purely by chance, then it is "Random."

Applying both kinds of partitions simultaneously, we obtain the following categories:

Intelligent Local Environments

An intelligent environment "knows our code." If a set of stresses is applied with both intelligence and control (local environment), then no code-based system is able to withstand such an attack. One must abandon at the outset any hope of defeating an intelligent local environment.

Random Local Environments

If a local environment is random rather than intelligent, then the problem of defeating the stress can be solved. One solution consists of simply providing enough equally likely alternatives so that the probability of picking the correct one at random is less than the specified safety level, $10^{-6}$. For a set of equally likely detonators, each of which must go off in order, we obtain

$$P\left\{\text{system failure}\right\} = 1/N! \leqq 10^{-6}. \qquad (9)$$

Solving for N,

$$1/N! \leqq 10^{-6} \qquad N=10 \text{ detonators}. \qquad (10)$$

While the size of this number is distressing, the possibility that a solution exists is gratifying!

Random Global Environments

This category of environment is not analyzed. Random global environments are covered by the next environmental category because of the worst-case hypothesis.

Intelligent Global Environments

This environment consists of one or more global stresses applied in the manner best designed to defeat the s/a strategy. The intelligent global environment is examined in the remainder of the report.

Quantification of Environments

For a simple s/a system, such as two detonators that must function with any order or timing to provide an output, the best way to defeat the s/a process is to make the environment as intense as possible. If the probability

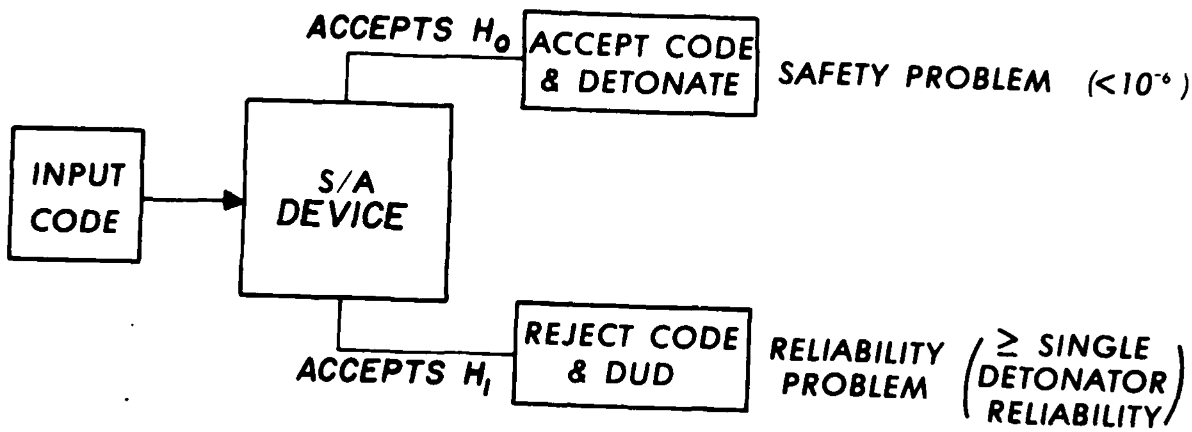


Figure 4. The S/A Decision

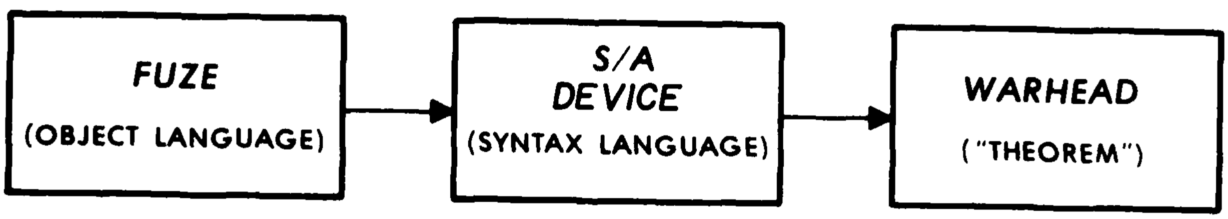


Figure 5. Safe/Arming Process as a Mathematical System

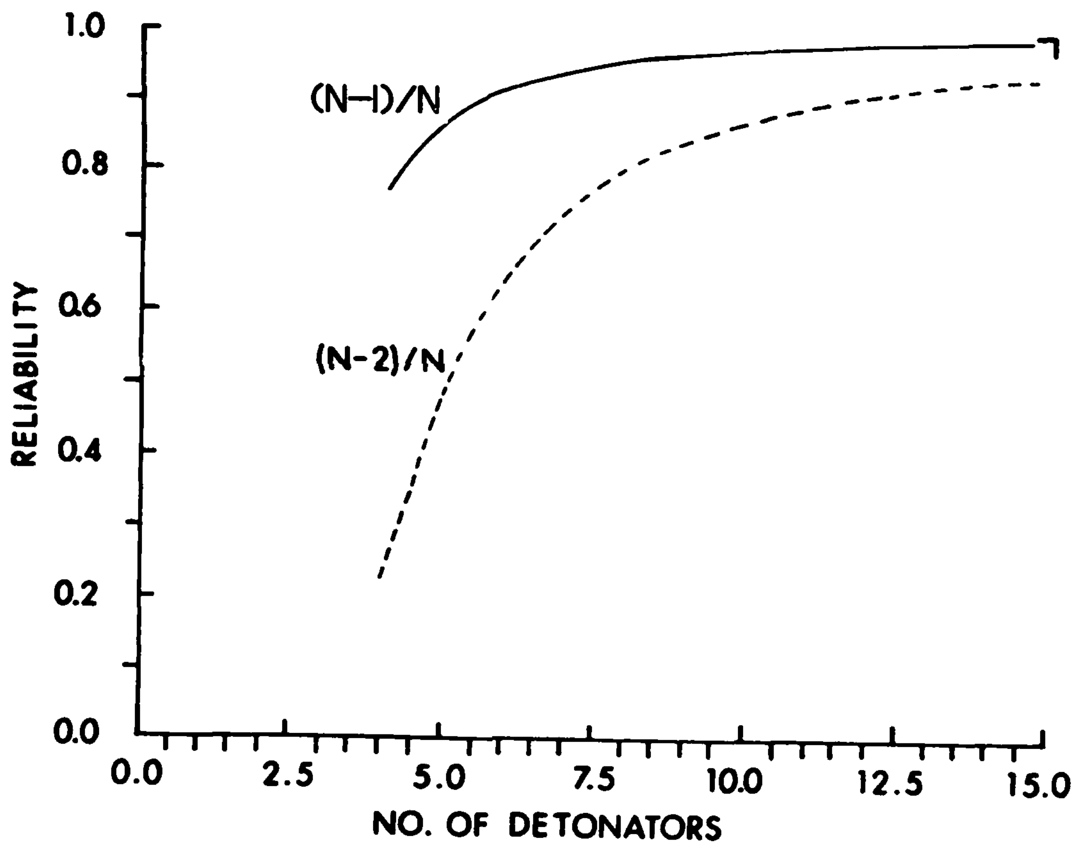


Figure 6. Detonator Reliability Needed to Meet System Criteria for [(N-1)/N] and [(N-2)/N] Systems

that any single detonator will fail is unity, then the simple s/a device is guaranteed to fail. Suppose, however, that the detonators must not only function, but they must function in a particular order and, perhaps, with a specified time between functions. A single, intense stress will not suffice to defeat this system with the same certainty as before. The more sophisticated system requires two distinct stresses properly sequenced. If the first stress is too intense, it may detonate both detonators and thus not yield a system event. It will be shown that the best way to attack such a system is to use a less stressful environment at the beginning and to progressively increase intensity until the last stress always makes the detonators function. Obviously, the worst possible stress on any particular system depends on the internal structure of the system, i.e., how the system interprets the environment it sees. In order to conduct a worst-case analysis it is necessary to consider the s/a process in an adversary relationship vis-a-vis its environment. The s/a process will have some specified decision-making structure (or strategy). The environment will attack the s/a strategy with a strategy of its own - a strategy that we structure to maximize the probability of producing a s/a defeat.

Since the optimal s/a strategy is precisely what we seek, it is productive to examine the problem in reverse order - find out what environments the s/a device may experience and then pick a s/a strategy to survive those environments. Specifying the "worst possible environment" for a s/a strategy can be facilitated by generalizing the concept of environmental stress. We would like to divorce the measure of environment from its physical description - that is, whether the stress arises from fire, electromagnetic pulse (EMP), etc. To do this, we pick some detonator as a standard. The sensitivity of the standard is, by definition, unity. Environmental stress intensity (E) can then be defined by

$$E = P\left\{\text{the standard detonator fails}\right\} . \qquad (11)$$

Equation (11) defines the intensity (measure) of environmental stress as a probability function - the probability that if the system to be tested exists at the time of the trial, and if the system to be tested is replaced by the standard detonator, then the standard detonator would fail the trial (detonator fires). One can immediately conceive of complications arising from this definition. What if the stress is spread out over a period of time? What if the s/a device is stressed again and again?

The first complication can be accommodated by introducing a distribution function F(t)

$$E = E(t_2) - E(t_1) = \int_{t_1}^{t_2} F(t)dt \; . \tag{12}$$

The functional form of F(t) will depend on the chosen environment. The second complication is really a statement that a system is often subjected to repeated trials. In a series of trials the history of previous results affects the outcome of any trial. This can best be handled by retaining the definition of Equation (11) and defining

$$\begin{aligned} T_2 &= E\left\{\text{second trial}\right\} \\ &= P\left\{\begin{matrix}\text{standard detonator fails} \\ \text{second trial given} \\ \text{it survived the first trial}\end{matrix}\right\} \; . \end{aligned} \tag{13}$$

The probability of the standard detonator surviving two successive trials would be

$$P\left\{\text{surviving both trials}\right\} = \left(1-T_1\right)\left(1-T_2\right) \; . \tag{14}$$

The concept of repeated trials is the basis for most environments. Using the *standard detonator concept*, we can begin to explore the kinds of environmental stress a system might experience. Stresses can arise from environments like fire (cookoff), temperature cycling, shock (mechanical or EMP), vibration, and even a modulated envelope of shock and vibration. The simplest class of stresses are those in which a single trial is described by a distribution function, F(t), as in Equation (12). If F(t) increases monotonically from time zero, then the stress models a cookoff-type environment, E1. The environmental stress type is labeled Ei, whereas the associated intensity is defined as $E_i$. If F(t) has some pulse-like structure, such as the normal distribution, then it models a temperature-cycle environment, E2. A stress of the E2 type which is very narrow, e.g., a normal distribution with a small $\sigma$, models a mechanical or electromagnetic (EMP) shock. If we take the limiting value of a decreasing $\sigma$

$$\lim_{\sigma \to 0} E2(t,\sigma) \to E3, \qquad \text{mechanical or EMP shock.} \tag{15}$$

More complicated environments can be composed of sequences of E2 and E3 stresses. Perhaps the most complicated would be a sequence of mixed E2 and E3 stresses of variable intensity and timing to form a modulated envelope of stresses.

Any safe/arm strategy must survive all of the environments. The worst-case hypothesis implies that the s/a device will be characterized by its performance against whichever system of stresses produces the lowest probability of survival. If $P_i$ is the probability of the s/a strategy failing environmental stress Ei, then the worst-case measure of s/a failure would be

$$M(s/a) = \text{maximum} \left\{ P_i \right\}, \tag{16}$$

where M(s/a) is the worst-case measure.

## VI. ANALYSIS OF SIMPLE SAFE/ARMING STRATEGIES

To analyze an environment that is intelligent and global, we seek to exercise the different types of stress, Ei, against whatever safe/arm strategies can be arrayed to resist the stresses.

We can identify five simple strategies as basic ones:

S1: TIMELESS: A specified number of detonators must function without regard to order or timing.

S2: SIMULTANEOUS: A specified number of detonators must function within some small time, t, of each other.

S3: SIMPLE ORDERING: (Sequential but not time-gated)* A specified set of detonators must function in proper order without regard to the timing between them.

S4: SEQUENTIAL: (Time-gated) Each detonator of a specified set must function in the correct order and at the proper time with respect to an absolute time standard.

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**The notation "time-gated" for strategies was suggested by D. Overman, US Army ERADCOM, Harry Diamond Laboratories, in a private communication.*

S5: SYNCHRONOUS: Each detonator of a specified set must function in the correct order and at the proper time with respect to a time standard established by one of the detonators. This differs from an S4 strategy because the time standard has the same uncertainty of function (jitter time) as the other system detonators. The time standard is generally chosen to be the first detonator to function that must function properly if the system itself is to function properly. In a [N/N] system, every detonator is needed; so the time standard is the first detonator. In a [(N-1)/N] system, one failure is permitted; so the time standard is the second detonator to function. In a [(N-k)/N] system, the time standard would be detonator number (k+1).

By biasing the timing of the detonators, a Synchronous strategy can be changed to a pseudo-S4:Sequential strategy. This has been confirmed by numerical analysis.* Consequently, analysis of synchronous systems is covered by analysis of sequential ones.

The S1: TIMELESS strategy is considered for mathematical rather than practical reasons. If an E1 type of stress is applied to a S1 system, then the probability of system failure can be made arbitrarily close to unity by simply increasing the stress. The S1 strategy is not viable for practical use because simple stresses like fire will defeat it.

The S2: SIMULTANEOUS strategy appears at first glance to be viable. If a stress of intensity E is used to exercise a two-detonator system using the S2 strategy, then the probability that both will fail is $E^2$. For an N-detonator system, where all N detonators must function simultaneously, the probability that all will function is $E^N$. Clearly, this looks like a good strategy.

The above argument is deceptive. Under the worst-case hypothesis we must assume that the intensity of the environmental stress can be raised as high as desired, including unity. When E=1, then all detonators are guaranteed to function. The safety of the s/a strategy then depends entirely upon whether or not the detonators function sufficiently close together to be considered simultaneous. This problem is analyzed in detail in Appendix A. The results are shown in Figure A-1, where the environmental stress width is scaled in terms of the width of the time-gate needed for detonators to be considered "simultaneous." The two curves show the performance of

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**W. Baker, System Engineering & Concepts Analysis Division, US Army Ballistic Research Laboratory, private communication.*

[(N-1)/N] and [N/N] systems, respectively, since these are the only ones of practical importance. The [(N-1)/N] system requires that at least (N-1) of the detonators be within the required simultaneity time-gate, while the [N/N] system requires that all N be within the time-gate. Each curve represents the stress width that can be tolerated by the system without failing a $10^{-6}$ safety requirement. What we see in Figure A-1 is that even a system with 15 detonators (the limit if a [(N-1)/N] system is to meet the reliability criteria), a stress about one time gate in width will defeat the s/a strategy. Since the time gate is of the order of ten micro-seconds, it is clear that the S2 strategy is vulnerable to shock. It is not a viable strategy because high shocks are common in military environments.

The S3: SIMPLE ORDERING strategy requires that detonators fire in a prescribed order. Although at first glance this strategy might seem inferior to the simultaneous strategy because it does not consider timing, simple order really contains more information than simultaneity. Since detonator timing is not significant, no timed environmental strategy has any advantage over another.

For an S3[N/N] strategy we can write

$$P\left\{\begin{matrix}\text{system}\\ \text{event}\end{matrix}\right\} = P\left\{S3\left[N/N\right]\text{fails}\right\} \tag{17a}$$

$$= P\left\{\begin{matrix}\text{all fail}\\ \text{in order}\end{matrix}\right\} = \left[\frac{1}{N!}\right] P\left\{\begin{matrix}\text{all fail}\\ \text{in any order}\end{matrix}\right\} \tag{17b}$$

$$= \frac{1}{N!} P\left\{S1\left[N/N\right]\text{fails}\right\} \tag{17c}$$

$$= \frac{1}{N!}\left[1\right] = \frac{1}{N!} , \tag{17d}$$

where P {all fail in any order} is exactly the probability function for the S1[N/N] case discussed under S1 of the previous page.

In order to meet the 1/million safety criterion

$$\left(1/N!\right) = 10^{-6} , \tag{18}$$

Thus

$$N \geqslant 10 . \tag{19}$$

For an [(N-1)/N] system, it is shown in Appendix B that

$$P\left\{\text{system event}\right\} = S\left\{N\right\}\left\{\frac{1}{N!}\right\} = \left[2N-3+S(N-1)\right]/N! \qquad (20)$$

for N > 2. S(2) = 2 by definition

which gives

$$N = 11: \qquad P\left\{\text{system event}\right\} = 2.5 \times 10^{-6} \qquad (21)$$

$$N = 12: \qquad P\left\{\text{system event}\right\} = 2.5 \times 10^{-7}. \qquad (22)$$

Thus, depending on one's conservatism, we need [10/11] or [11/12] to satisfy the safety requirement.

The S4: SEQUENTIAL (Time-Gated) strategy should be more efficient than a simple ordering. The requirement that all detonators necessary to system function must function within specified time channels with respect to the chosen time standard eliminates many environmental stresses that would defeat an S3 strategy of the same number of detonators. The proof in Appendix C shows that intuition is correct. The method used in Appendix C consists of two general steps: First, the S4 [N/N] problem is solved. Then the S4 [(N-1)/N] problem is written in terms of the [N/N] case by using the binomial expansion formula. The [(N-1)/N] case is then solved by differentiating the terms of the expansion with respect to each variable. The resulting equations are evaluated numerically.

The results are summarized in Equations (23):

$$S4\left[6/7\right] = 5.2 \times 10^{-5} \qquad (23a)$$

$$S4\left[7/8\right] = 3.4 \times 10^{-6} \qquad (23b)$$

$$S4\left[8/9\right] = 1.8 \times 10^{-7}. \qquad (23c)$$

Examining these simple strategies it is clear that only the S3: Simple Ordering and the S4: Sequential strategies are of practical utility in designing in-line s/a systems if the worst-case safety and reliability criteria are to be met. A surprising result is that time-gating only saves three detonators over an S3 [11/12] strategy.

## VII. APPLICATION OF THE ANALYSIS TO SOME PROPOSED IN-LINE SAFE/ARMING DESIGNS

Much of the development of coded-detonator S/A hardware technology has originated at the US Army, Harry Diamond Laboratory (HDL), Adelphi, Maryland. Three of the four devices analyzed in this section are taken from designs shown in a report[1] on their work in this field. The fourth is a generic design currently being investigated at BRL and earlier at the Naval Surface Weapons Center (NSWC), Dahlgren, Virginia. In applying the analysis to these designs, two main points must be acknowledged. First an objection has been raised to the application of a worst-case hypothesis to the safe/arm problem,* since a worst-case assumption is not representative of munition life-cycle experience. This question is not frivolous. Underspecification of a s/a mechanism results in a dangerous munition, but overspecification wastes resources and unnecessarily results in fewer munitions for the user. A fair and complete discussion of the issue is beyond the scope of this report. Let us simply note that the results of the analysis are, indeed, based on the worst-case assumption. A second point that must be noted is that the devices modeled in the analysis do not reflect other than obvious improvements, such as changing the confluent shock device from an S2 to an S4 strategy and may not represent current technology. The s/a designs analyzed are shown in Table 1.

TABLE 1. TYPES OF IN-LINE S/A DEVICES

| | |
|---|---|
| Explosive Bolt Device (Explosive Barrier Module) | -HDL |
| Simple Explosive Logic Device | -HDL |
| Confluent Shock Device | -HDL |
| Synchronous Explosive Logic Device | -NSWC/BRL |

### A. Explosive Bolt S/A Device

Shown in Figure 7, the Explosive Bolt device consists of a base (generally plastic), three explosive motors (A,B,C), a lead cup (d), and a detonator (D). The substrate is cast or machined to provide the cavities and slots shown in Figure 7. The slots in the substrate define three interlocking "bolts,"

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[1] *R.K. Warner and D.L. Overman, "Explosive Train Technology for Electronic Fuzes," HDL-PR-71-1, November 1971.*

**D. Overman, US Army ERADCOM, Harry Diamond Laboratory, Adelphi, MD. Letter dated 7 July 1980.*

also labeled A,B,C. The device is a mechanical implementation of a Simple Ordering strategy. The bolts are arranged so that each must be moved in sequence in order to move the third bolt, the one containing the detonator, into line. Bolt A has a tab that locks bolt B so that bolt B cannot be moved until after bolt A slides enough to unlock the tab. Bolts A and B together prevent bolt C from moving until they have been removed. Thus, only the sequence A,B,C will permit bolt C to slide into position. This sequence is performed by firing the explosive motors A,B,C in proper order. Once this has been accomplished, then bolt C slides detonator D into line with lead cup,(d). Firing the detonator, D, in proper sequence following motors A,B,C will produce normal functioning of the in-line s/a device. Any firing out of sequence will produce a dud. The bolts cannot be sequenced instantaneously as in a mathematical ordering, but the timing can be within a millisecond (est.). The firing strategy can be approximated as S3 [4/4].

B. Simple Explosive Logic S/A Device

This s/a device consists of a substrate made of plastic or some similar inert material in which small, rectangular channels are molded or cut. These channels form a computing network that performs the s/a logic. The network is shown diagrammatically in Figure 8. The inputs (from detonators) are labeled A,B,C in the diagram. Any of the types of logic gate shown in the rest of Figure 8 might be used. Each null-gate consists of a signal channel and a control channel. If a detonation in the control channel reaches the intersection before any detonation in the signal channel, then the control detonation will destroy the intersection and thus "cut off" the signal detonation. Functionally, the null-gate behaves as a "break" switch. In Figure 8 the initiation points are labeled A,B,C in order of their proper firing. The intersections, where logic switching occurs, are labeled 1 through 6. Operation of the s/a device is as follows:

If detonation from A reaches intersections 4 and 5 before their respective signal detonations, then 4 and 5 will be cut.

If detonation from input B then occurs, it will not be able to pass intersection 5 since the cutting detonation from A preceded it. This will prevent the control detonation B from cutting intersection 2. Consequently, the detonation from B will proceed along the signal path through intersection 2. It will then proceed to intersection 6 where it will cut the intersection.

Finally, a detonation from input C is received. Since A has previously cut intersection 4, the signal path from C can pass through intersection 3 (it has not been cut). The detonation then advances to intersection 6. Input B has previously cut the gate at 6, so C cannot detonate the control channel at intersection 1. The detonation from C can thus proceed along the longer signal channel, through intersection 1 and into the output lead. Any failure to detonate in the proper order results in a dud. The firing strategy is S3 [3/3].

## C. Timed Dual Pulse S/A Device

Although developed as a non-explosive shock transfer device, the dual pulse shock pyramid has been proposed for use as an in-line s/a device by itself, on the theory that the likelihood that two detonators would fire in almost perfect simultaneity is sufficiently remote that the device is safe.

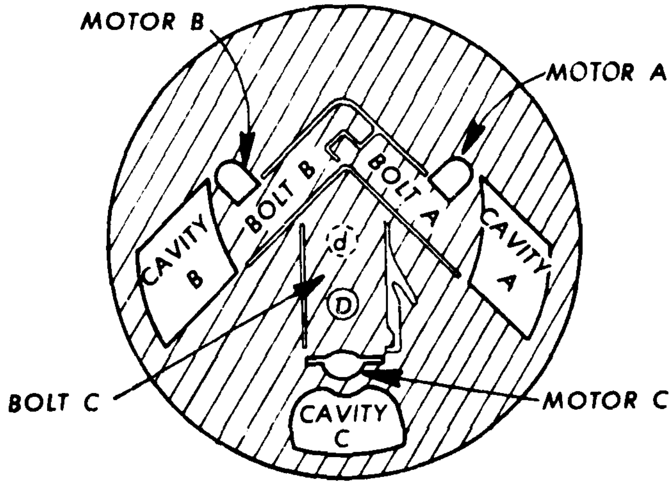
The Timed Dual Pulse s/a device is shown in more detail in Figure 9. The s/a device is composed of three main elements: a donor explosive, an inert barrier in the form of a wedge or pyramid, and an acceptor explosive.

The attenuation of the inert barrier is engineered so that if the donor explosive detonates accidentally and only a single shock front passes over the barrier, then the pressure transmitted to the acceptor explosive is not sufficient to initiate detonation in it. If two detonation fronts collide precisely in front of the inert barrier, then the collision will produce a much higher (nearly double) pressure in the acceptor explosive and will initiate detonation in the acceptor. Functionally, the device is an AND gate. If the inert barrier is fabricated with more than two sides - a multi-sided pyramid rather than a wedge - then the device will function as a many-input AND gate. Variations in construction, such as using a directed slug instead of reflected shocks, have been successfully tested. This is an S2 [N/N] strategy.

As shown in section VI, the simultaneous strategy is not viable. The Timed Dual Pulse s/a strategy can be changed from an S2 to an S4: SEQUENTIAL one by making each leg of the donor explosive a different length or otherwise making each leg so that detonations initiated at different times will collide over the barrier. The timing strategy of the Dual Pulse s/a device becomes: S4 [N/N].

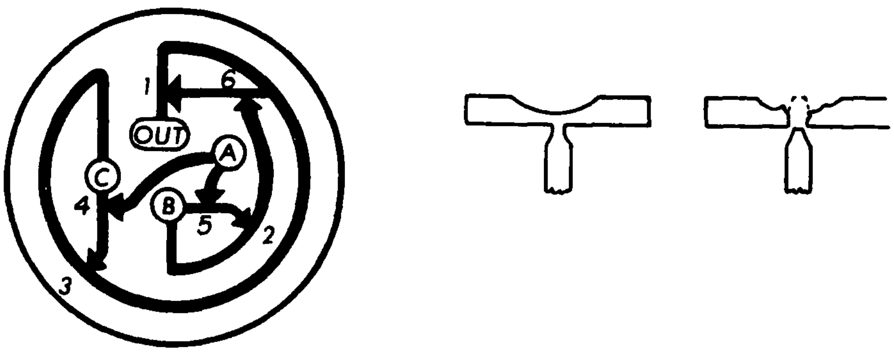
## D. Synchronous Explosive Logic S/A Device

This device is based on the "time window" concept. At the proper time, a "window" is opened. This is done by an explosive logic network called the "clock." The window is held open by sending the clock output through an explosive delay path. When it exits the delay path, the clock detonation enters another explosive logic network called the "decoder." In the decoder, the clock detonation is compared with the inputs from other detonators. If the correct number of detonator inputs has been received, then the clock pulse provides an output to the warhead. Otherwise, the s/a device produces a dud. Although the decoder operates on an S3 [(N-k)/N] strategy, this can be converted to an S4 [(N-k)/N] by the same method - varying the length of input legs - that was used to convert the Timed Dual Pulse device from S2 to S4. A functional block diagram of the Synchronous Explosive Logic s/a device is shown in Figure 10.



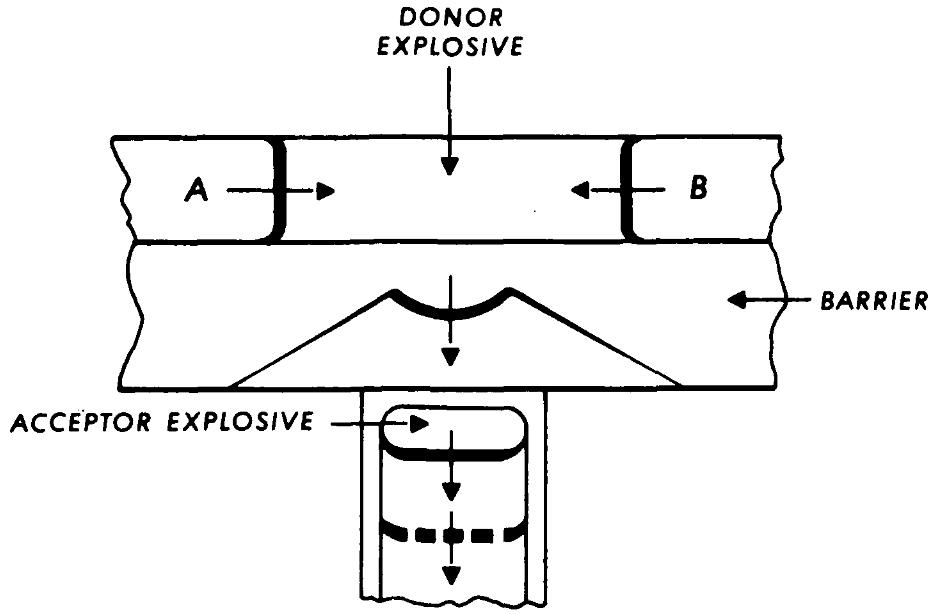

FUNCTION MODE: A THEN B THEN C THEN D
*STRATEGY*: *S3* [4/4]

Figure 7. Explosive Bolt Device



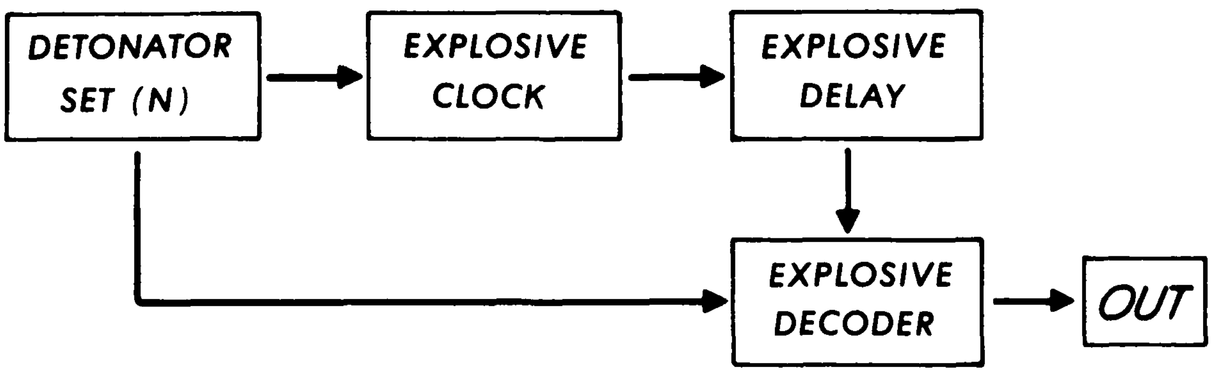

FUNCTION MODE: A THEN B THEN C
*STRATEGY*: *S3* [3/3]

Figure 8. Simple Explosive Logic Device




FUNCTION MODE: A & B SIMULTANEOUSLY
STRATEGY: S4 [2/2]

Figure 9. Confluent Shock (Timed Dual Pulse) Device




FUNCTION MODE:

ANY (N-K) OUT OF N DETONATORS.

EACH MUST FUNCTION AT ITS PROPER TIME.

STRATEGY: S4 [(N-K)/N]

Figure 10. Synchronous Explosive Logic Device

## VIII. SUMMARY OF RESULTS

It is important to note again that the results of Section VII apply only to the devices as they were modeled. Some designs may be capable of improvement to accommodate the safety requirements of various s/a strategies, while others may not. Such potential growth was no factor in the analysis. With this caveat explicitly stated, the results are shown in Table 2.

TABLE 2. IN-LINE DEVICE STRATEGIES

| DEVICE | STRATEGY |
|---|---|
| Explosive Bolt | S3[4/4]* |
| Simple Explosive Logic | S3[3/3]* |
| Timed Dual Pulse | S4[2/2]* |
| Synchronous Explosive Logic | S4[(N-k)/N] |

*It is assumed that all of these devices could be improved by increasing the number of system variables.

## IX. CONCLUSIONS

### A. Specific Conclusions

By assuming that a s/a device will experience the worst possible stress, we have developed a quantitative approach to analyzing safe/arm devices.

Of the four simple strategies examined (S1-S4), two, the S1: Timeless and the S2: Simultaneous are not suitable for use in safe/arm design.

Both the S3: Simple Ordering and the S4: Sequential strategies can meet s/a safety requirements even with the worst-case assumption. The S4 strategy requires fewer detonators than the S3 strategy.

Detonator efficiency alone does not mean that S4 is a superior strategy to S3. S4 devices require a time standard. This is provided by an explosive logic network called a "clock." The need for a clock means that S4 networks are more complex than S3 networks. S3-based strategies have a second advantage: Since the time between detonators is not very important (only the order), uniformity of function time is not very important. Cheaper detonators can be used in an S3-based system. By electrically

timing detonator firing signals far enough apart, irregularities in function time would be cancelled. Increased time between firing pulses has an added bonus - power demand on the missile electrical supply is less.

Strategies that use the [(N-1)/N] function class are favored over those that use the [N/N] class because the small increase (1 detonator) in input variables required by the [(N-1)/N] class is more than compensated by the inherent reliability advantages of permitting at least one detonator to malfunction. Strategies using lower function classes, e.g., [(N-2)/N], will not be favored because the number of detonators required to meet safety requirements becomes prohibitive.

Four proposed in-line s/a devices were examined using the quantitative methodology. The Explosive Bolt and Simple Explosive Logic devices, as modeled in the analysis, did not meet the requirements of the worst-case assumption, but it appears that either one could be expanded to meet the safety requirement. Since both devices use the [N/N] class strategy, reliability penalties would occur. If a change to the [(N-1)/N] class became necessary, then complete redesign might be needed.

The Timed Dual Pulse, if improved to S4 [8/8] and Synchronous Explosive Logic (depending on one's conservatism) S4 [7/8] devices, can meet the safety requirements, but the Timed Dual Pulse device may have difficulty meeting the reliability standards because its strategy is based on the [N/N] class.

## B. General Conclusions

Safe/arm devices can be designed to insure absolute immunity from any global environmental stress with a failure rate of no more than one per million.

The fact that even in-line and stored-energy devices can achieve such immunity constitutes absolute, quantitative proof of the technological feasibility of in-line and stored-energy s/a designs.

The worst-case hypothesis has been criticized as not representative of genuine munition life-cycle experience. As stated, this objection is absolutely correct. However, the worst-case assumption does give valuable insight into the response of s/a designs to their environment. The assumption also provides guaranteed performance where experimental or experiential data are absent. Since stored-energy and in-line devices are inherently less robust, they must be evaluated more conservatively than traditional s/a designs. Obviously, a quantitative theory that accurately modeled munition life-cycle experience would be much better as an evaluation tool than the worst-case procedure. Such a life-cycle model would test precisely what we want the s/a device to do. A precise statement of what we want a s/a device to do has not been defined. This is clearly needed before quantitative techniques can be fully developed.

# APPENDIX A.

## THE MULTIPLE INDEPENDENT EVENT MODEL OF THE SIMULTANEOUS STRATEGY

## APPENDIX A.

## THE MULTIPLE INDEPENDENT EVENT MODEL
## OF THE SIMULTANEOUS STRATEGY

We wish to analyze the behavior of a system of detonators in which all, or some specified subset, must function "simultaneously." Since the function time of detonators is not perfectly predictable, we must, for reliability's sake, consider detonators that function within some time, t, of the mean time to function as being "simultaneous." The problem is further compounded by the use of one of the detonators as a time standard. Even worse, if we let any detonator be a time standard, there are multiple possibilities for "simultaneity." This multiple time standard results in a very complicated problem.

Let us first make several simplifications:

(1) We will pick only one detonator to be a time standard and all other detonators will be referred to it.

(2) All detonators, including the time standard, are picked from the same population and the deviations from the mean function time of the population are normally distributed about that mean.

(3) All detonators are independent of each other, that is, the function time of one detonator does not influence the function time of any other detonator.

The implication of simplification #1 is that we will compute a system probability of function that is smaller than that of a system with multiple time standards, so that if we examine the response of the system to an accidental stress, the computed response will be a lower bound, so that if the simplified system fails, then a real system must also fail.

The implication of simplification #2 is that we can pick a single number, $\delta = t$, such that if any detonator is within $\delta$ of the time standard, then it is "simultaneous."

The implication of simplification #3 is that we only have to consider the relation of each detonator with the time standard. The system function probabilities can thus be computed from the simultaneity measurement of each detonator w.r.t. the time standard.

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This can be written formally by defining:

$t$ is the time measured by some absolute time standard.

$e_o$ is the event, "The local time standard functions."

$z_o$ is the time that the local time standard functions, with the mean time of functioning being at $t = 0$.

$e_i$ is the event, the i-th detonator functions within a time, $\delta$, of the time standard, $z_o$.

$z_i$ is the time that the i-th detonator functions.

All times, $z$, are normally distributed about the time $t = 0$.
Thus, we can write

$$P\left\{e_i\right\} = P\left\{ |z_o - z_i| \leq \delta \right\}. \tag{A-1}$$

Obviously,

$$P\left\{e_o\right\} = 1 \tag{A-2}$$

$$P\left\{e_i e_j\right\} = P\left\{e_i\right\} \; P\left\{e_j\right\} \quad i \neq j . \tag{A-3}$$

This means that each detonator event is independent of the other detonator events. If we have a set of events, $\{e_i\}$, and event probability, $P\,\{e_i\} = q$ for all $i$, then

$$P\left\{\text{ALL events, } e_i \text{ must occur}\right\} = q^N \quad \text{where N is the number of events.} \tag{A-4}$$

$$P\left\{\text{at MOST one of the events, } e_i \text{ is not true}\right\} = q^N + Nq^{N-1}\left(1-q\right). \tag{A-5}$$

The first equation defines the "N/N" case, the second equation defines the "(N-1)/N" case. Similar equations can be developed for other cases such as "at most two detonators fail" by using additional terms of the binomial expansion, but only the [N/N] and [(N-1)/N] cases are of practical interest.

Now suppose that we have a system of N detonators like the system described above. Suppose that the system is subjected to some environmental stress such that the probability that any given detonator will function due to the stress is normally distributed about a mean time, T=0, with standard deviation, sigma. We must note that this environmentally induced detonator function distribution is completely different from the function time distribution during normal operation - the $\delta$ discussed above. Normal operation of the system is fixed by the physical characteristics of detonator construction, while the environmentally induced distribution can be as wide or as narrow as nature desires. Thus, $\delta$ is fixed by the physical limitations of the detonators we use, while $\sigma$ can vary as nature (the environment) wills.

We know that as the sigma of the environmental stress becomes smaller, the probability of system function (the specified subset of N detonators functioning within $\delta$ of the standard) will rise. At some value of $\sigma$, the system probability of function due to environmental stress will just equal $10^{-6}$.

The simplest system we can look at is an [N/N] one, e.g., all the detonators must function properly to cause a system event. Since the detonators are independent events, we can write

$$P\left\{\text{system event}\right\} = A^N = 10^{-6} \tag{A-6}$$

where A is a value to be determined once we know N. Somewhat more complicated is the [(N-1)/N] case, where no more than one detonator can fail to function within the simultaneity requirement if we are to have a system event. Since the detonators are independent of each other, we can use the binomial expansion

$$P\left\{\text{system event}\right\} = A^N + NA^{N-1}\left(1-A\right) = 10^{-6} \quad . \tag{A-7}$$

For [(N-2)/N] or other systems, we obtain a polynomial in A from the binomial expansion by including the extra terms needed. A represents the probability that any one detonator fires within the time window defined as simultaneous. All needed information about system strategy is contained in the equation defining A. The problem reduces to an investigation of how the

value of A and the $\sigma$ of the environmental stress are related. This problem has been examined in three different ways.

FIRST APPROACH

The first approach is pessimistic because we replace the environmental normal distribution by a uniform distribution of width $\pm\ \sigma$, where $\sigma$ is the standard deviation of the (now replaced) environmental normal distribution. This is shown schematically in Figure A-2(a). The height of the new distribution is $1/2\sigma$, so the distribution is more clustered about the mean than the original normal distribution. The area under the distribution is unity.

Next, let us partition the distribution into vertical strips of width $2\delta$, with each strip having area

$$\left(2\delta\right)\left(\frac{1}{2\sigma}\right) = \text{Area} \quad \text{where} \left(\frac{1}{2\sigma}\right) \text{ is the height .} \tag{A-8}$$

A single strip is simply the probability that a detonator will fail within $\pm\delta$ of the time standard.

We require that the probability that any detonator fail simultaneously with the standard must equal A, so

$$2\delta\left(\frac{1}{2\sigma}\right) = A \tag{A-9}$$

rearranging,

$$\sigma = \frac{\delta}{A} \ . \tag{A-10}$$

SECOND APPROACH

The second approach is due to W. Baker.* It is less pessimistic than the first solution, but it is still a worst-case analysis for the normal distribution. As in the first solution, let us replace the normal distribution of the environment with a uniform distribution. Instead of fixing the width at $\pm\sigma$, however, let's fix the height equal to the value of the normal distribution at its mean, $\ell_o$. This constitutes precisely the worst case for a normally distributed environment, since the normal distribution is never greater than its value at the mean. This schematic is shown in Figure A-2(b). The width of the uniform distribution will be slightly more than $\pm\sigma$, say it is $\pm\ c\sigma$, where $c > 1$.

*Private communication.

As before, if we divide the uniform distribution into strips $2\delta$ wide, we can equate the area of a single strip to A.

This time, however, the strip is not $1/2\sigma$ high. Instead it is $\ell_o = 1/2c\sigma$ high. Thus, we can write

$$2\delta\left(\frac{1}{2c\sigma}\right) = A \tag{A-11}$$

rearranging,

$$\sigma = \delta/c\,A\ . \tag{A-12}$$

THIRD APPROACH

The third approach to the problem is due to M. Taylor.*

Instead of trying to replace the environmental distribution with a simpler one, let's consider the expression

$$P\left\{ |z_i - z_0| \leq \delta \right\} = A, \text{ where } z_i \text{ is the function time of the i-th detonator.} \tag{A-13}$$

The left side of this equation is the probability that two normally distributed function times (the time standard and another detonator) will function within $\pm\ \delta$ of each other. The corresponding diagram for this case is shown in Figure A-2(c).

The right side of equation (A-13) is, of course, the value we wish this probability to have. The above equation can be rewritten

$$P\left\{ -\delta \leq (z_i - z_0) \leq \delta \right\} = A\ . \tag{A-14}$$

This equation can be transformed into one with a variable that is normally distributed with mean of zero and variance of one

$$P\left\{ -\frac{\delta}{\sqrt{2}\sigma} \leq \frac{z_i - z_0}{\sqrt{2}\sigma} \leq \frac{\delta}{\sqrt{2}\sigma} \right\} = A\ . \tag{A-15}$$

Since the normal distribution is symmetrical about the mean

$$P\left\{ 0 \leq \frac{z_i - z_0}{\sqrt{2}\sigma} \leq \frac{\delta}{\sqrt{2}\sigma} \right\} = A/2\ . \tag{A-16}$$

---

**M. Taylor, System Engineering & Concepts Analysis Division, US Army Ballistic Research Laboratory, private communication.*

From this we obtain

$$\frac{\delta}{\sqrt{2}\sigma} = Z\left(\frac{A}{2}\right), \tag{A-17}$$

where Z(A/2) is the number of standard deviations needed to enclose an area of A/2 from the mean to $\delta/\sqrt{2}\sigma$ under the standard normal distribution.

Let's compare the three solutions. The first is a worst-case for a distribution somewhat more peaked than the normal distribution. The second is the worst-case for the normal distribution. Third is the exact solution which gives the expected results for a large number of trials, assuming the environment is normally distributed. Each solution yields an equation involving the three variables, $\sigma, \delta$, and A.

$$\text{Solution I:} \quad \sigma = \frac{\delta}{A}\ ; \tag{A-18a}$$

$$\text{Solution II:} \quad \sigma = \frac{\delta}{cA} \quad \text{where c is constant} > 1\ ; \tag{A-18b}$$

$$\text{Solution III:} \quad \sigma = \frac{\delta}{\sqrt{2}\,Z\left(\frac{A}{2}\right)}\ . \tag{A-18c}$$

These three solutions will be used to obtain an answer to one of the most important questions we can ask about an S2 system, "Just how large a system is needed to satisfy safety requirements?" To answer this question, consider first the general polynomial expression for the quantity, A,

$$f(A) = 10^{-6}\ . \tag{A-19}$$

This is an N-th order polynomial in A. The two simplest cases are [N/N] and [(N-1)/N], which give, respectively,

$$A^N = 10^{-6} \tag{A-20}$$

and

$$A^N + NA^{N-1}(1-A) = NA^{N-1} - (N-1)A^N = 10^{-6}\ . \tag{A-21}$$

When N is small, e.g., N=1, then A must be small.

$$A^1 = 10^{-6} \qquad \text{implies that } A = 10^{-6} \tag{A-22}$$

for the [(N-1)/N] case there is no solution below N=2,

$$2A^{2-1} - (2-1)A^2 = 10^{-6} \qquad \text{(A-23)}$$

$$2A - A^2 = 10^{-6} \quad \text{implies that } A = 5.00001 \times 10^{-7}\,. \qquad \text{(A-24)}$$

Case #1: [N/N] strategy.

As N gets very large:

$$A^N = 10^{-6} \qquad |0 \leqslant A \leqslant 1| \qquad \text{(A-25)}$$

has different solutions for A that seem to tend toward a value A=1 as N tends to infinity. We want to show that as N tends to infinity, A does indeed approach 1. In the general case (A less than 1),

$$A^N = h, \quad \text{where h is a positive number} \leqslant 1. \qquad \text{(A-26)}$$

It is necessary to first show that the limit function exists as N tends to infinity. This can be done by using the Cauchy condition for uniform convergence;[A-1] the Cauchy Theorem requires that functions in an infinite sequence get "closer" as N gets larger.

We must show that for every $\epsilon > 0$, there exists an N such that m,n, > N implies

$$|f_m - f_n| < \epsilon \qquad \text{for every A in } [0,1]\,. \qquad \text{(A-27)}$$

In our case: we wish to show

$$|A^m - A^n| < \epsilon \qquad \text{for every A in } [0,1]\,. \qquad \text{(A-28)}$$

Proof:

For every m,n, positive, $A^m$, $A^n$ are positive since A is positive.

If m,n, are greater than N, $0 < A^m, A^n < A^N$;

---

A-1 *T.M. Apostle, Mathematical Analysis, pp. 395, Addison-Wesley Co., 1957.*

therefore,

$$|A^m - A^n| < A^N, \qquad \text{let } A^N = \epsilon, \text{ say.} \tag{A-29}$$

then for all m, n > N, $| A^m - A^n | < \epsilon$ which completes the proof.

Since we know that the limit exists we can solve for the value of A

$$A = h^{\frac{1}{N}} \rightarrow 1 \quad \text{as } N \rightarrow \infty . \tag{A-30}$$

Thus, for the [N/N] case, the limit value of A is 1.

Case #2: The [(N-1)/N] strategy.

The [(N-1)/N] case also yields a Cauchy Sequence of functions which we can handle by building on the result obtained from the [N/N] case. The [(N-1)/N] equation is obtained from the first two terms of the binomial expansion $(A+(1-A))^N$. For the [(N-1)/N] case we can write

$$A^n + nA^{n-1}\left(1-A\right) = b . \tag{A-31}$$

We wish to explore

$$\lim_{n \rightarrow \infty}\left\{A^n + nA^{n-1}\left(1-A\right)\right\}. \tag{A-32}$$

By the sum of the limits theorem

$$\lim_{n \rightarrow \infty}\left[A^n + nA^{n-1}\left(1-A\right)\right] = \lim_{n \rightarrow \infty} A^n + \lim_{n \rightarrow \infty} nA^{n-1}\left(1-A\right) \tag{A-33}$$

if both limits exist.

We know from the previous [N/N] case that the first term has a limit for all $A \varepsilon [0,1]$. We need only find the limiting case (if it exists) for the second term, $nA^{n-1}(1-A)$.

Consider the sequence of functions

$$f_n = nx\left(1-x\right)^n . \tag{A-34}$$

This is very nearly the same as the sequence of functions that we seek to evaluate, with x=1-A.

It is shown in Apostle[A-2] that this sequence has a limit function as n tends to infinity. The limit function is defined over the closed interval, [0,1] and has the value 0 for each point in the interval. That is

$$\lim_{n\to\infty} f_n = \lim_{n\to\infty} nx\left(1-x\right)^n = 0 \quad \text{for all} \quad x \in \left[0,1\right] . \tag{A-35}$$

Substitute y=1-x to get

$$\lim_{n\to\infty} f_n\left(y\right) = \lim_{n\to\infty} n(1-y)y^n = 0 \qquad y \in [0,1] . \tag{A-36}$$

Now, consider

$$\frac{f_n\left(y\right)}{y} = \left(\frac{1}{y}\right)n\left(1-y\right)y^n \qquad y \neq 0 . \tag{A-37}$$

Using the Cauchy condition for uniform convergence, we must show that for all $\epsilon > 0$, there exists N such that m,n > N imply

$$\left|\frac{f_m}{y} - \frac{f_n}{y}\right| < \epsilon , \tag{A-38}$$

or

$$\left|\frac{1}{y}m(1-y)y^m - \frac{1}{y}n(1-y)y^n\right| < \epsilon ; \tag{A-39}$$

but, we know that $y \in (0,1]$, so

$$\left|\frac{f_m}{y} - \frac{f_n}{y}\right| = \frac{1}{y}\left|f_m - f_n\right| . \tag{A-40}$$

The term $(1/y)n(1-y)y^n$ converges to a limit function that has value 0 throughout the half open interval (0,1], but

$$\frac{1}{y}n\left(1-y\right)y^n = ny^{n-1}\left(1-y\right), \tag{A-41}$$

which is exactly what we seek, with y substituted for A.

---

A-2 *Apostle, pp. 391, Example 1.*

Thus

$$\lim_{n \to \infty} nA^{n-1}\left(1-A\right) = 0 \quad \text{for all } A \in \left[0,1\right]. \tag{A-42}$$

We don't care that the limit point at A=0 has been lost, since we are exploring in the neighborhood of the upper limit point, A=1.

Since the limit exists over (0,1], and the limit for $A^n$ also exists over that interval, then the sum of the limits theorem applies; and the limit function

$$A^n + nA^{n-1}\left(1-A\right) = b \tag{A-43}$$

also exists in the half open interval, (0,1]. The limit value of A → 1 as n (and therefore N) → ∞.

A similar argument can be extended to any finite [(N-k)/N] case. Each new term is obtained by taking the next term of the binomial expansion. For any [(N-k)/N] system, the limit value of A, as the number of detonators gets very large, is thus A=1.

Comparison of the three solutions for the S2 Strategy

We can use the result of Eq. (A-42) to re-examine the three solutions to the problem of how wide an environmental pulse is needed to defeat the system:

$$\text{Solution I: } \sigma = \frac{\delta}{A}\ ; \tag{A-44a}$$

$$\text{Solution II: } \sigma = \frac{\delta}{cA} \quad \text{where c is a constant} > 1\ ; \tag{A-44b}$$

$$\text{Solution III: } \sigma = \frac{\delta}{\sqrt{2}\, Z\left(\frac{A}{2}\right)}\ . \tag{A-44c}$$

As shown above, for large N the value of A converges to 1, so

$$\text{Solution I: } \sigma = \delta\ ; \tag{A-45a}$$

$$\text{Solution II: } \sigma = \frac{\delta}{c} \quad ; \tag{A-45b}$$

$$\text{Solution III: } \sigma = \frac{\delta}{\sqrt{2}\, Z\left(\frac{A}{2}\right)} \to 0 \text{ as } A \to 1 \, . \tag{A-45c}$$

The first two solutions indicate that a finite environmental pulse width, as shown in Figure A-1, will defeat even an infinitely large system. The third solution, one we may feel is more "precise," confirms the intuitive feeling that the $\sigma$ of the environment needed to defeat a system must tend to zero as the number of detonators tends to infinity. Clearly, there is at least the appearance of a contradiction among the different solutions.

Instead of examining the mathematically infinite case, suppose we examine a "practical" infinity - suppose

A/2 = .4998, say. This corresponds to a system of about 40,000 detonators. From the tabulated Normal Curve of Error:

Z (.4998) corresponds to an average of 3.54 standard deviations. If we use this value in Solution III,

$$\sigma = \frac{\delta}{\sqrt{2}(3.54)} \cong \frac{\delta}{5} \, . \tag{A-46}$$

The apparent contradiction is resolved. While the environmental $\sigma$ does tend to zero for infinite N, it converges so slowly that even huge values of N will be defeated by fairly wide environmental pulses. A theoretical solution to do this same problem has been published by W. Baker and M. Taylor.[A-3]

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[A-3] *W. E. Baker and M. S. Taylor, "An Order Statistic Approach to Fuze Design," ARBRL-TR-02313, April 1981 (AD A100753).*

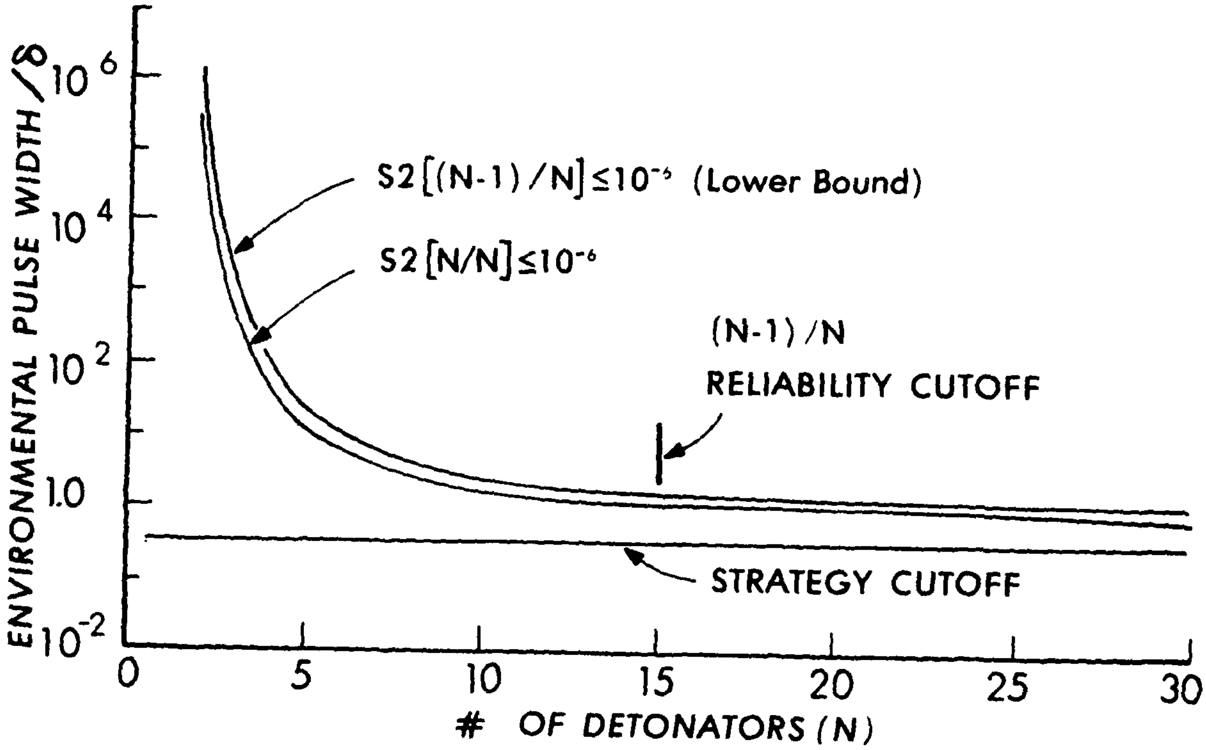
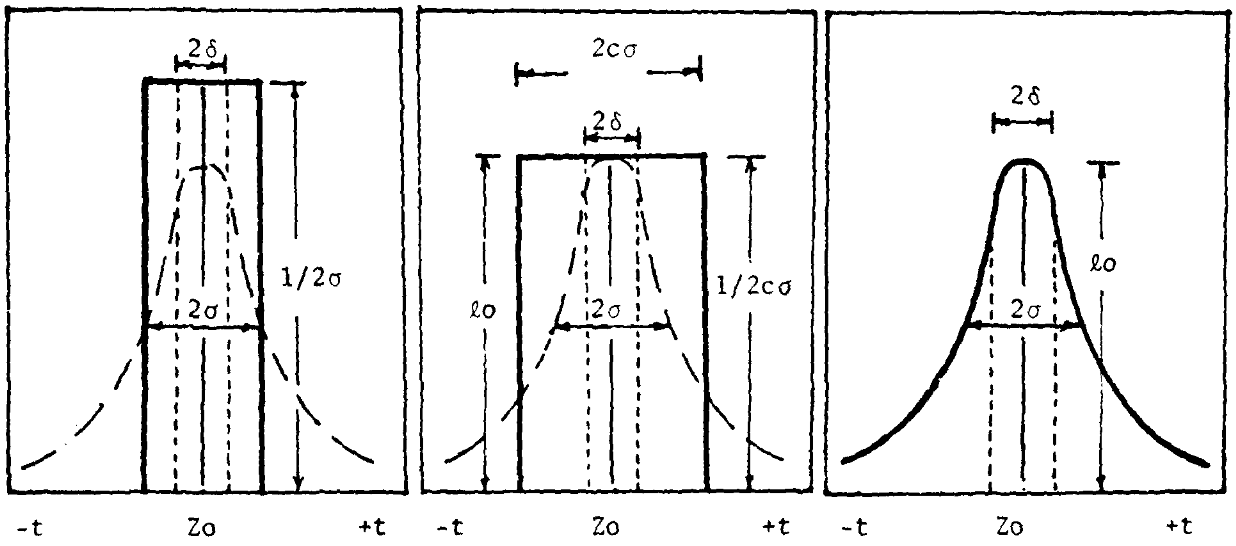



Figure A-1. Simultaneous Strategy
S2[N/N]; S2[(N-1)/N]




(a) Uniform distribution of width $\pm\sigma$.

(b) Uniform distribution of width $\pm c\sigma$.

(c) Normal distribution with standard deviation $\sigma$.

Figure A-2. Illustrations of Approximations to and Exact Normal Distribution Representing the Environmental Stress.

## APPENDIX B.

## A FORMULA FOR THE COMPUTATION OF S3 [(N-1)/N] STRATEGIES

## APPENDIX B.

### A FORMULA FOR THE COMPUTATION OF S3 [(N-1)/N] STRATEGIES

Consider a system of N detonators in which at least (N-1) must fire in proper order. Denote the set of all outcomes which produce an explosive event by S(N). We can partition the set of all possible outcomes into three mutually exclusive, collectively exhaustive sets of events which we can call Class I, Class II, and Class III. Each class can be evaluated separately and then the results summed to get the total of all possible outcomes which will yield a system event.

Class I consists of sequences in which detonator #1 fires first. Since detonator #1 is supposed to be first, this is no failure. Of the remaining N-1 detonators only N-2 must fire in proper sequence to make the s/a device fail. This is an S3 [(N-2)/(N-1)] strategy with outcomes denoted as S(N-1).

Class II consists of those sequences where detonator #2 fires first. Since one failure has been experienced, no other failures are permitted in the remaining N-1 detonators. This can only occur if the detonators 3,4,5,...,n are sequenced in that order no matter when detonator #1 fires. The possible sequences yielding this are: 1,3,4,...; 3,1,4,...; 3,4,1,5,...; etc. There are exactly N-1 of these.

Class III events consist of those outcomes in which some detonator other than #1 or #2 fires first. There are exactly N-2 detonators which can fit this criterion. For each of them, there is exactly one sequence which will yield an explosive event: K,1,2,3,4,...,(K-1), (K+1),...,n. There are thus exactly N-2 outcomes in Class III.

If we sum the contributions of Classes I, II, and III, we get

$$\begin{aligned} S(N) &= \text{Class I} + \text{Class II} + \text{Class III} \\ &= S(N-1) + N-1 + N-2 \\ &= S(N-1) + 2N - 3. \end{aligned} \tag{B-1}$$

This is a recursive definition for N > 2. S(2) is defined as 2. The cumulative results for detonator systems with up to 12 detonators are shown in Table B-1.



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TABLE B-1.

FAILURE PROBABILITIES FOR VARIOUS S3 [(N-1)/N] STRATEGIES

| N | 2N-3 | + | S(N-1) | = | S(N) | P(N) = S(N)/N! |
|---|---|---|---|---|---|---|
| 2 | - | | - | | 2 | 1.00 |
| 3 | 3 | | 2 | | 5 | .83 |
| 4 | 5 | | 5 | | 10 | .42 |
| 5 | 7 | | 10 | | 17 | .14 |
| 6 | 9 | | 17 | | 26 | .04 |
| 7 | 11 | | 26 | | 37 | .007 |
| 8 | 13 | | 37 | | 50 | .001 |
| 9 | 15 | | 50 | | 65 | $1.8 \times 10^{-4}$ |
| 10 | 17 | | 65 | | 82 | $2.3 \times 10^{-5}$ |
| 11 | 19 | | 82 | | 101 | $2.5 \times 10^{-6}$ |
| 12 | 21 | | 101 | | 122 | $2.5 \times 10^{-7}$ |

Thus the 1/million safety criterion is met by an S3 [11/12] strategy.

# APPENDIX C.

## FORMULAE FOR AN S4: SEQUENTIAL STRATEGY

## FORMULAE FOR AN S4: SEQUENTIAL STRATEGY

Consider a system of N detonators, each physically identical, but each assigned a proper time to function. We wish to compute the system probability of function when the system is subjected to a sequence of trials by a global environmental stress and the environmental strategy is maximized. We wish to determine what that maximal environmental strategy is. Only 'look-shoot' strategies are examined, that is, the environment first 'looks' at the s/a device then 'shoots' the maximal strategy determined from the look. More complex strategies of the 'look-shoot-look-...' type might be superior in some circumstances. Some analysis of them has been completed, but will not be covered in this report. A complete coverage of flexible (look-shoot-look-shoot-...) strategies may be examined in a later report.

It seems clear that the safe/arm device will maximize its chances of surviving environmental stress by requiring that exactly one detonator function properly in each appropriate time window and that the environment will improve its chances by timing the stresses to precisely match the detonators' proper times to function.

Case #1: The number of stresses equals the number of detonators. The detonator strategy class is S4[N/N].

Let $P_o$ be the system probability of function. Let $p_k$ be the probability that a detonator will function on the k-th stress given that it exists on the k-th stress. Since all detonators are physically identical, all detonators that exist at the time the k-th stress is applied will have equal probability of functioning. In a time-gated strategy the proper detonator should function on the k-th stress, while those supposed to function after the k-th (and have not gone off before the k-th stress) are NOT to function. Let $p_k$ be continuous and differentiable in the interval [0,1]. Let k, N be finite integers with $1 \leq k \leq N$. Thus, the system probability of function can be written

$$P = \prod_{k=1}^{N} p_k\left(1-p_k\right)^{N-k} . \qquad \text{(C-1)}$$

If $P_o$ has an extremum in the N-dimensional interval (0,1), it is necessary that

$$\frac{\partial P_o}{\partial p_k} = 0 \text{ for EACH } k . \qquad \text{(C-2)}$$

We can separate the variables if we let

$$f_k\left(p_k\right) = p_k\left(1-p_k\right)^{N-k} \qquad \text{(C-3)}$$

so that

$$P_o = f_1 f_2 f_3 \ldots\ldots f_n . \qquad \text{(C-4)}$$

The necessary condition of equation (C-2) is equivalent to

$$f_k' = 0 \cdot \tag{C-5}$$

Solving this we get

$$f_k' = (1 - p_k)^{N-k} - (N-k)p_k(1-p_k)^{N-k-1} = 0 ; \tag{C-6}$$

therefore,

$$(1 - p_k)^{N-k-1} [1 - p_k - (N-k) p_k] = 0 \cdot \tag{C-7}$$

Aside from the non-maximal solution $p_k=1$,

$$p_k = \frac{1}{N-k+1} \cdot \tag{C-8}$$

The functions of equation (C-8) are single valued, so that only one extremum exists for each $p_k$. It is easy to show that each of the functions $f_k$ is concave downward, and that (C-8) defines a maximum for $P_o$.

We note that 1) $f_k > 0$ when $p_k$ equals $1/(N-k+1)$;

2) $f_k(0) = f_k(1) = 0$ for $k < N$.

But equation (C-8) insures that there is only one extremum on $f_k \in [0,1]$, so the functions $f_k$ must have maximum value at $p_k = 1/(N-k+1)$. Because the variables are separable, the probability function $P_o$ has a maximum when all $f_k$ have maxima. This means that the optimum stress sequence for the environment to use to defeat a safe-arm system using the sequential strategy is $\{1/N, 1/(N-1), \ldots, 1/1\}$.

If we use these values for $p_k$ in equation (C-1), then we get

$$P\left(\text{at optimum environment}\right) = \prod_{k=1}^{N}\left(\frac{1}{N-k+1}\right)\left(1 - \frac{1}{N-k+1}\right)^{N-k} . \tag{C-9}$$

If we set the result of equation (C-9) equal to the safety standard

$$P = \prod_{k}\left(\frac{1}{N-k+1}\right)\left(\frac{N-k}{N-k+1}\right)^{N-k} \leqslant 10^{-6} . \tag{C-10}$$

Numerical iteration can be used to give

$$P\left\{S4\left[7/7\right]\right\} = \frac{1}{7^7} = 1.2 \times 10^{-6} . \tag{C-11}$$

We can use the formula for the S4[N/N] to obtain a solution for the S4[(N-1)/N] strategy. For notational convenience let $P_{\frac{N-i}{N-j}}$ denote P{S4[(N-i)/(N-j)]}. Assume that the number of stresses equals the number of detonators. The detonator strategy class is S4[(N-1)/N]:

$$P_{\frac{N-1}{N}} = P\left\{\begin{matrix}\text{all N function}\\\text{properly}\end{matrix}\right\} + P\left\{\text{exactly one malfunctions}\right\} . \qquad \text{(C-12)}$$

The first term in Equation (C-12) is just the result for the [N/N] strategy, as shown in Equation (C-9). The second term can be written

$$P\left\{\text{exactly one malfunctions}\right\} = \sum_{i=1}^{N} P_{\frac{N}{N}} \times \frac{P\left\{\text{i-th det malfunctions}\right\}}{P\left\{\begin{matrix}\text{i-th det functions}\\\text{properly}\end{matrix}\right\}} . \qquad \text{(C-13)}$$

Let $S_i$ be the probability that the i-th detonator functions properly. Then, equation (C-12) can be written

$$P_{\frac{N-1}{N}} = P_{\frac{N}{N}} + P_{\frac{N}{N}} \sum_{i=1}^{N} \frac{1 - S_i}{S_i} . \qquad \text{(C-14)}$$

Simplifying,

$$P_{\frac{N-1}{N}} = P_{\frac{N}{N}} \times \left(1 - N + \sum_{i=1}^{N} \frac{1}{S_i}\right) . \qquad \text{(C-15)}$$

Now $S_i$ is the probability that the i-th detonator works properly. This probability can be expressed as the probability that the i-th detonator functions on the i-th stress <u>IF it exists on the i-th stress</u> times the probability that it WILL exist on the i-th stress:

$$S_i = p_i \prod_{j=0}^{i-1} \left(1 - p_j\right), \quad \text{where } p_0 = 0 . \qquad \text{(C-16)}$$

Using this expression in equation (C-15) gives

$$P_{\frac{N-1}{N}} = P_{\frac{N}{N}} \times \left[1 - N + \sum_{i=1}^{N} \frac{1}{p_i \prod_{j=0}^{i-1} \left(1 - p_j\right)}\right] . \qquad \text{(C-17)}$$

As before, the optimum environmental strategy is obtained by differentiating Equation (C-17) and setting each derivative equal to zero:

$$\frac{\partial}{\partial p_i}\left[P_{\frac{N-1}{N}}\right] = 0 \ . \tag{C-18}$$

Using a modified Newton-Raphson optimization routine, this problem was solved numerically for three systems: [6/7], [7/8], and [8/9]. The results were:

$$S4\left|\frac{6}{7}\right| = 5.2 \times 10^{-5} \ ; \tag{C-19a}$$

$$S4\left|\frac{7}{8}\right| = 3.4 \times 10^{-6} \ ; \tag{C-19b}$$

$$S4\left|\frac{8}{9}\right| = 1.8 \times 10^{-7} \ . \tag{C-19c}$$

<u>Case #2</u>: The number of pulses exceeds the number of detonators. Detonator strategy is S4[N/N]. If the environment attempts to produce a resonant responsefrom a system, then the strategy can be defeated by spacing the detonator channels (time windows) so that the time spacings are prime relative to one another (measured in time window widths). This forces the environment to generate an excessive number of stress pulses. The extra stresses must evoke no system response, since any detonator functioning between time windows would be mistimed. The number of extra trials is given by

$$\#\text{ of extra trials} = \sum_i^N T_i - N \ , \tag{C-20}$$

where $T_i$ is the spacing of the i-th time window and N is the number of detonators. The number of extra trials can be made arbitrarily large, so that the probability of the system surviving the extra trials intact is then arbitrarily small. This makes the constant frequency attack non-viable.

The final environmental option considered is a group-pulse attack. In this approach the environment attempts to fail the s/a device with the first group of pulses. If the system neither functions nor duds, then a second group of pulses tests the system. The process continues until the system either functions or duds. If the system survives a given set of pulses unscathed, then it is clear that whatever environmental strategy was optimal for the just completed group of stresses will be optimal for the next group. It follows, therefore, that the system function probability for an infinite set of pulse groups would be

$$P = P_0(1 + R + R^2 + R^3 \ldots), \tag{C-21}$$

where $P_o$ is the same probability function defined in Equation (C-1) and R is the probability that a group of N pulses fails to produce any system response at all,

$$R = \prod_{k=1}^{N} \left(1-p_k\right)^N . \tag{C-22}$$

Note that $1+R+R^2+ \ldots$ is an infinite geometric series. Using the formula for the sum of a geometric series in Equation (C-21) we obtain

$$P = \frac{P_0}{1-R} . \tag{C-23}$$

Once again we examine

$$P' = \partial\frac{P}{\partial p_k} = 0 . \tag{C-24}$$

It is convenient to note that

$$P_0' = \partial\frac{P_0}{\partial p_k} = P_0\left[\frac{1}{p_k} - \frac{N-k}{1-p_k}\right] \tag{C-25}$$

and

$$R' = \partial\frac{R}{\partial p_k} = -\frac{N R}{1-p_k} . \tag{C-26}$$

Using Equations (C-25) and (C-26) in Equation (C-23) we obtain the general result

$$P' = \frac{P_0}{1-R}\left[\frac{1}{p_k} - \frac{N-k}{1-p_k}\right] - \frac{P_0}{(1-R)^2}\left[\frac{N R}{1-p_k}\right]. \tag{C-27}$$

Solving for $p_k$,

$$p_k = \frac{1-R}{N-(k-1)(1-R)} . \tag{C-28}$$

Although no general proof is given here, there is a general solution which shows that pulse groups after the first are identically zero. The work has been done by a mathematician in the United Kingdom who plans to publish his proof separately.

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